

Why the safe interest rate cannot summarize financing conditions

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Integral Economics — Research note
29 September 2026

Abstract

The safe return averages household discount factors. It need not summarize the cost of providing incentives across aggregate states. We give an exact conditional-price comparison with identical technology, wealth and opportunity capacity, establish global optimality over high and low effort, and derive the whole wealth frontier. Investment scale and outside LP capital are reported separately.

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The result at a glance

With equally likely aggregate states, $p = 4/5$, $p_0 = 2/5$, $Y = 3$, private benefit $b = 2/5$, wealth $W = 1/10$ and capacity $\bar{D} = 1$, the discount-factor pairs $(1/2, 1/2)$ and $(3/5, 2/5)$ both imply $R = 2$. Their optimal total investment scales are $1/2$ and $5/6$, and their GP values are $2/5$ and $2/3$. These are supplied-price contracting environments with linear terminal utility and high effort selected at incentive equality.

1 Fundraising at household state prices

1.1 Why a scalar return is insufficient

The GP's acquisition decision depends on what its proceeds can finance in the next vintage. The relevant input is usable wealth. LPs participate in that next vintage only if its promised payoff compensates them at their consumption values and leaves the manager willing to supply effort. Under aggregate risk, compensating the manager in one state can be more expensive to LPs than compensating it in another. That substitution is lost by averaging state prices into a safe return.

Consider one successor launch, conditional on the public market state. There are finitely many future states s , with probabilities $\pi_s > 0$ and positive household discount factors m_s . Saving is available at gross $R = (\sum_s \pi_s m_s)^{-1}$. The GP has cash W and finite capacity $0 \leq \bar{D} < \infty$ supplied by its feasible opportunities. Scale D pays $Y_s D$ on success and zero otherwise. High and low effort succeed with probabilities p_s and $p_{0s} < p_s$; low effort gives terminal private benefit bD . Success-contingent GP compensation is z_s . The GP contributes w and saves $W - w$. Its terminal preferences are linear in money and the specified effort benefit.

Proposition 1.1 (Stochastic fundraising program). *Define $G(W)$ as the optimum of*

$$\max_{D, w, z} \sum_s \pi_s p_s z_s - R w \quad (1)$$

subject to

$$0 \leq D \leq \bar{D}, \quad 0 \leq w \leq W, \quad w \leq D, \quad (2)$$

$$0 \leq z_s \leq Y_s D \quad \text{for every } s, \quad (3)$$

$$\sum_s \pi_s (p_s - p_{0s}) z_s \geq bD, \quad (4)$$

$$\sum_s \pi_s m_s p_s (Y_s D - z_s) \geq D - w. \quad (5)$$

The optimal terminal GP value under high effort or saving is $J(W) = RW + G(W)$. The optimum is attained. J is continuous, concave and piecewise affine in $W \geq 0$. Its incremental franchise G is nonnegative, nondecreasing and concave; $J(W + h) \geq J(W) + Rh$ for $h \geq 0$. Expanding opportunity capacity weakly raises G .

Proof. Zero scale, contribution and compensation are feasible. All constraints are linear and closed. Capacity bounds D ; $w \leq \min\{W, \bar{D}\}$ and $z_s \leq Y_s \bar{D}$ bound all remaining variables. Hence the feasible polyhedron is compact and the linear objective attains its maximum. Saved wealth plus expected success compensation is precisely RW plus (1). The effort difference gives (4); LPs price their residual success receipts in (5). Thus the program describes the stated contract domain exactly.

For $W_\lambda = \lambda W_1 + (1 - \lambda)W_2$, any convex combination of feasible contracts at W_1, W_2 is feasible at W_λ . The wealth inequality is the only wealth-dependent constraint, and its right side has the same combination. Applying this construction to optimizers proves concavity of G , and adding linear RW proves concavity of J . Feasible sets expand with W , so G is nondecreasing and the displayed lower bound on J follows. Zero scale proves $G \geq 0$.

There are finitely many possible linear-program bases. On each parameter interval for which a basis is feasible and optimal, its objective is affine in W ; the finitely many interval boundaries give a piecewise-affine value. Finite concavity ensures continuity for $W > 0$. For a sequence $W_n \downarrow 0$, optimizers lie in one common compact box. Any subsequential limit is feasible at zero and has the limiting objective by continuity. Feasible-set inclusion and optimality therefore give right-continuity at zero. Finally, larger $\bar{D}$ expands the constraints' feasible set. $\square$

The theorem's capacity input is economically distinct from its wealth input. It does not claim that $\bar{D}$ stays fixed when routing, household values or entry change. Here opportunity capacity is supplied to the conditional problem. Nor is a kink in personal usable wealth a statement about LP assets under management.

Proposition 1.2 (A global effort check). *Let $A_0 = \sum_s \pi_s p_{0s} Y_s$ and $m_{\max} = \max_s m_s$. If*

$$A_0 + b \leq \frac{1}{m_{\max}}, \quad (6)$$

then every limited-liability contract that induces low effort gives the GP at most RW . Consequently Proposition 1.1 is globally optimal over both effort recommendations in the stated domain. High effort is selected at incentive indifference.

Proof. If ℓ denotes nonnegative LP receipts, participation implies $\mathbb{E}[m\ell] \geq D - w$. Since $m \leq m_{\max}$, $\mathbb{E}\ell \geq (D - w)/m_{\max}$. Output and private benefit under low effort sum to $(A_0 + b)D$ in expectation. Adding saved GP wealth and subtracting LP receipts bounds GP utility by

$$R(W - w) + (A_0 + b)D - \frac{D - w}{m_{\max}} = RW + \left(A_0 + b - \frac{1}{m_{\max}}\right)D + \left(\frac{1}{m_{\max}} - R\right)w \leq RW.$$

The first coefficient is nonpositive by (6); the second is nonpositive because $R = 1/\mathbb{E}m \geq 1/m_{\max}$. Saving is feasible in the high-effort-or-saving program. Therefore no low-effort contract improves its optimum. $\square$

Condition (6) is sufficient, not necessary. Strict effort can be implemented where feasible by giving a positive rent slack. Replacing an exact optimum at equality by that limit must be reported explicitly; the stated contract uses effort at indifference.

1.2 The constant-price slice

Corollary 1.3 (Wealth capacity and outside LP capital). *Suppose $m_s = 1/B$, $p_s = p$, $p_{0s} = p_0$, $Y_s = Y$, and set*

$$A = pY, \quad P = p \left(Y - \frac{b}{p - p_0} \right).$$

Under $0 \leq P < B < A$ and $p_0Y + b \leq B$, total optimal scale and GP value are

$$D^* = \min \left\{ \bar{D}, \frac{BW}{B - P} \right\}, \tag{7}$$

$$J(W) = BW + (A - B)D^*. \tag{8}$$

Below $W^\dagger = \bar{D}(1 - P/B)$ all GP wealth is invested and outside LP capital is $PW/(B - P)$. At capacity, total scale is unique, but optimal GP contribution can be any

$$w \in [\bar{D}(1 - P/B), \min\{W, \bar{D}\}].$$

Proof. IC bounds expected pledgeable LP receipts by PD , so participation requires $B(D - w) \leq PD$. Because $w \leq W$, any feasible scale obeys $D \leq BW/(B - P)$. Conversely, at any scale up to this bound and capacity, choose a contribution at least $D(1 - P/B)$ and at most $\min(W, D)$. Give LPs success receipts $B(D - w)/p$ and the GP the residual. These are nonnegative, conserve output and satisfy IC. LP participation binds. Combining expected GP compensation and savings gives $BW + (A - B)D$. Since $A > B$, the largest feasible scale is optimal. Proposition 1.2 rules out profitable low effort. Below capacity the scale bound forces $w = W$; at capacity the displayed interval is exactly the set of feasible implementing contributions. $\square$

This is the familiar limited-pledgeability mechanism (Holmström and Tirole, 1997, *Quarterly Journal of Economics*, 112(3), 663–691). It makes the difference between total commitments and external LP commitments transparent. Above capacity, extra GP wealth can replace LP

capital without reducing total investment. No strict external-fundraising comparison follows solely from a strict increase in GP value.

At fixed physical technology, the rent per unit in launch-good units is $\eta(B) = A/B - 1$, and

$$\eta'(B) = -A/B^2, \quad \frac{d}{dB} \frac{B}{B-P} = -\frac{P}{(B-P)^2}.$$

Both terms matter for a benchmark comparison. Endogenous prices and search opportunities also change $A, P, \bar{D}$ and the distribution of W , so these derivatives alone do not sign a market-wide intervention.

1.3 A proved failure of scalar compression

Proposition 1.4 (Same safe return, different fundraising). *Let two future states be equally likely. Set $p = 4/5$, $p_0 = 2/5$, $Y = 3$, $b = 2/5$, $W = 1/10$ and $\bar{D} = 1$. With $m = (1/2, 1/2)$ the optimum has $(D, J) = (1/2, 2/5)$. With $m = (3/5, 2/5)$ it has $(D, J) = (5/6, 2/3)$. Both state-price vectors imply the same safe gross return $R = 2$, and both satisfy the global low-effort bound.*

Proof. For constant prices, $A = 12/5$ and $P = 8/5$; Corollary 1.3 gives the first pair. For the stochastic prices, IC requires $z_L + z_H \geq 2D$, and LP participation is

$$\frac{6}{25}z_L + \frac{4}{25}z_H \leq \frac{1}{5}D + w.$$

Its left side is at least $8D/25$, so $3D/25 \leq w \leq 1/10$ and $D \leq 5/6$. Multiplying that participation inequality by $5/2$ also gives

$$\frac{2}{5}(z_L + z_H) - 2w \leq \frac{1}{2}D + \frac{1}{2}w - \frac{1}{5}z_L \leq \frac{7}{15}.$$

Choose $D = 5/6$, $w = 1/10$, $z_L = 0$, $z_H = 5/3$. Every constraint holds and the last bound is attained. Adding saved-wealth baseline $RW = 1/5$ gives $J = 2/3$. Finally, $A_0 + b = 8/5$ is below both 2 and $5/3$, the respective reciprocal maximum discount factors. $\square$

The effect comes from shifting the manager's incentive compensation into the cheaper state. The safe rate cannot record that substitution. This is a conditional comparison of state prices, not yet a pair of fully reoptimized general equilibria. Its role in the integrated proof is precise: any macro interface passing only the safe rate discards a variable that the actual successor LP program uses.

1.4 Fundraising curvature and compensation reallocation

The stochastic program has more structure than a single capacity kink. For the two-state primitives of Proposition 1.4, the entire value function is

$$J(W) = \min \left\{ \frac{20}{3}W, \frac{5}{2}W + \frac{1}{2}, 2W + \frac{16}{25} \right\}. \quad (9)$$

Its kinks are $W = 3/25$ and $W = 7/25$. The first reaches the opportunity cap. Between the kinks scale is already one, but more usable wealth permits a different incentive-compensation allocation. Only after the second kink is the incremental franchise constant.

Proof. Write $Z = z_L + z_H$. The LP constraint is $.24z_L + .16z_H \leq .2D + w$, and IC gives $Z \geq 2D$. Thus $D \leq 25w/3$. The inequality used in Proposition 1.4 gives $G \leq D/2 + w/2 - z_L/5$. With $w \leq W$ and $D \leq 1$, this gives both $G \leq 14W/3$ and $G \leq 1/2 + W/2$. A third bound follows directly by substituting $w \geq -.2D + .24z_L + .16z_H$ into $G = .4Z - 2w$:

$$G \leq .4D - .08z_L + .08z_H \leq .64D \leq .64.$$

Adding $2W$ proves the three upper bounds in (9).

For $0 \leq W \leq 3/25$, choose $D = 25W/3$, $w = W$, $z = (0, 50W/3)$. For $3/25 \leq W \leq 7/25$, choose $D = 1$, $w = W$, $z = (0, (25W + 5)/4)$. Above $7/25$, choose $D = 1$, $w = 7/25$, $z = (0, 3)$. Each allocation satisfies cash, limited liability, IC and LP participation; direct substitution attains the corresponding lower branch. These intervals exhaust $W \geq 0$, proving the formula globally. $\square$

2 Outside capital and the illustrated contracts

At $W = 1/10$ both optimizers invest all GP wealth, $w = 1/10$. Thus outside LP capital $D - w$ is respectively $2/5$ and $11/15$, distinct from total investment D . In the constant-price environment choose $z_L = z_H = 1/2$. In the state-dependent environment choose $z_L = 0$, $z_H = 5/3$. These amounts are success-contingent compensation, not unconditional transfers. The second state has the lower household cost $m_H = 2/5$.

The constant-price wealth frontier follows from the scalar corollary:

$$J_{\text{constant}}(W) = 2W + \frac{2}{5} \min\{1, 5W\}.$$

The first state-dependent kink $3/25$ reaches the physical opportunity cap. The second $7/25$ exhausts the compensation reallocation gain at that cap. All plotted quantities follow from the displayed formulas.

Source and version

This presentation extract uses the programme's *Fundraising at household state prices*, Propositions "Stochastic fundraising program", "A global effort check" and "Same safe return, different fundraising", and the exact frontier in *Computations that test the common construction*. The required programme and arguments are reproduced above; no other file is required to use this note.