

How financial contracts direct capital toward entrepreneurs

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Abstract

Financial offers direct entrepreneur applications when they are backed by both genuine matching capacity and cash. In a conserved-capacity institution with a funded matched-surplus frontier, the globally optimal effective founder rent is $r^*(u) = \sqrt{\ell u W / c}$. Continuous participant-cost distributions yield a unique positive market-clearing outcome. The note includes the information and funding construction that makes the surplus frontier attainable.

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Economic scope and notation

Private clues arrive after routing. Founders and incumbent owners retire at current closing; their preferences are current net cash minus true information costs. LPs precommit and fund their worst-case calls. W in this note denotes total matched surplus, not GP cash; ℓ denotes venue capacity per funded slot, $c = \chi\ell/(1 + \ell)$ a matching parameter, r a founder's matched utility net of testing, and u gross application utility before creation cost. The matching parameter is chosen so the meeting probabilities are at most one (as in the source, $0 < \chi \leq 1$ suffices). A menu cannot manufacture physical capacity, and physical capacity cannot replace financial reserves.

1 Optimal information with funded pretest insurance

A public history without an authentic certificate need not be uninformative: a privately informed report can itself reveal a test result. An optimal certificate gate therefore requires an argument about the full information available to the parties. The following institution permits that argument and pays for every no-finance contingency.

1.1 The financial and information domain

Consider a matched opportunity with primary cost K . If viable, it pays the actual two-payout venture claim, worth Q at the household prices; if unviable it pays zero. Conditional on viability, aggregate output risk is independent of the clue and test coins. The founder, incumbent and originating buyer reach their last personal consumption date at current closing, have zero initial financial wealth, and have no bequest motive. They can market all retained venture rights at the actual publicly assessed prices. Liabilities require full backing and final redemption or legal transfer. There is no other productive right or unsecured credit against a future career.

These primitives imply that true optimized utility is current net cash minus nonpecuniary costs. An owned future claim can be sold; retaining its post-retirement payout gives no utility. Consolidating collateralized trades at the positive pricing functional cannot add current cash without surrendering an owned asset or funding a liability. A nonnegative terminal estate is worthless and has nonnegative cost. This argument covers actual allowed current hedges and sales. The inequality $m > \delta$ alone would not give the same conclusion for a longer-lived insider who privately disagrees with a public claim valuation. This is an explicit terminal career replacement of that richer problem.

The shared private clue gives viability probability $\alpha_s \in [\underline{\alpha}, \bar{\alpha}] \subset (0, 1)$. Founder and incumbent sequentially choose observable tests before either result arrives. Test j costs $c_j > 0$ in current utility and, conditional on viability, yields an authentic good-only certificate with probability $\eta_j \in (0, 1)$. The two coins are conditionally independent. Founder publication precedes incumbent publication; withholding is feasible and fabrication is impossible. Additional private reports and separate consent are permitted. Any pretest message generated only from the shared clue can be pooled, without a uniqueness claim.

Households commit *before private testing* to an LP contract covering both information insurance and the venture. They advance an actual worst-case reserve, receive unused funds back at closing, and own the future output of a financed venture. Participation is ex ante under the actual signal/strategy law. A later LP veto after a failed test is not part of this commitment. Menus may otherwise use arbitrary continuous claims, funded transfers, primary-investment lotteries, reports, exclusion and refusal. A financed opportunity uses its full K ; a lottery can choose its continuation probability. There is no compulsory authentication gate in this larger choice domain.

1.2 An optimal gate over continuous menus

Theorem 1.1 (Insured information and optimal certification). *In the stated matched institution suppose*

$$\bar{\alpha}Q < K < Q, \quad \underline{\alpha}\eta_j(1 - \eta_k)(Q - K) > c_j \quad (j \neq k). \quad (1)$$

Put $S = Q - K$, $\bar{\alpha} = \mathbb{E}\alpha_s$, and $\rho = \bar{\alpha}[1 - (1 - \eta_F)(1 - \eta_I)]$. Suppose $B^ = \rho S - c_F - c_I \geq 0$ and the actual household reserve can cover $K + \rho S$. Then the maximum expected buyer payoff over all feasible continuous menus and their consistent sequential follower assessments is*

$$B^* = \rho(Q - K) - c_F - c_I. \quad (2)$$

A committed menu attains it: pay insiders c_F, c_I and the buyer B^ whenever both observable tests are performed and the parties consent; finance exactly when at least one authentic certificate is published. The stipends are paid even when both tests fail. Both tests and truthful publication are selected weak best responses at this maximum. Their social surplus advantage in (1) is strict.*

Proof. First consolidate actual claim sales, transfers, collateral, LP calls and refunds. Every insider's true utility is current cash, and no outside resource enters. Ex ante LP participation

bounds total expected insider fees plus buyer payoff by $\mathbb{E}[(\theta Q - K)x]$, where $x \in [0, 1]$ is the actual continuation decision or probability. Law of iterated expectation gives this bound whether information is publicly certified, reported, concealed or revealed by consent. No-finance insurance payments remain on its left side. Since each insider can decline participation for zero, its expected fee covers its expected test cost. Hence buyer payoff is at most

$$\mathbb{E}[(\theta Q - K)x - c_F e_F - c_I e_I]. \quad (3)$$

Now condition on the complete private clue and results of tests actually taken. If a certificate exists, viability is certain and primary surplus is $S > 0$. Otherwise its posterior is

$$\frac{\alpha_s \prod_j (1 - \eta_j e_j)}{1 - \alpha_s + \alpha_s \prod_j (1 - \eta_j e_j)} \leq \bar{\alpha},$$

so financing has strictly negative conditional surplus. Messages, reports and votes depend only on this profile and randomization; they cannot create more information than this full profile. Thus its optimal primary rule finances exactly profiles containing a certificate. For fixed test choices the maximal conditional surplus is

$$\alpha_s \left[1 - \prod_j (1 - \eta_j e_j) \right] S - c_F e_F - c_I e_I.$$

Adding test j raises it by $\alpha_s \eta_j (1 - \eta_k e_k) S - c_j > 0$ under (1), including when the other test is already active. Both tests therefore maximize this bound at every clue. Averaging gives (2); lotteries cannot improve it.

In the proposed menu a founder skip removes both-test compensation, so the incumbent skips its costly unremunerated test. After a founder test, the incumbent can test for net utility zero or skip for zero; select testing. The founder likewise tests for zero instead of skipping for zero. Publication does not change the fixed stipend; select authentic publication when available. Unsupported reports are ignored. Final consent protects positive compensation. These sequentially optimal strategies and actual Bayes beliefs form a consistent assessment; finite completely mixed trembles approach their indifferent choices and the signal likelihood gives the limiting beliefs.

Let $R = K + \rho S \leq Q$ be the actual pretest reserve. At a realized history the call is $C = K \mathbf{1}\{\text{published certificate}\} + \rho S \leq R$. Households receive refund $R - C$ now. Its mean net call is $\rho K + \rho S = \rho Q$, exactly the price of the whole output from the financed ventures. Even the failed-test stipends are paid from that already advanced reserve. Participation binds, inputs are purchased once, and every inequality in the upper bound is attained. $\square$

2 Directed offers with conserved matching capacity

A separate market for every continuously chosen offer does not inherit the usual competitive-search interpretation from the original bounded matching formula. This section proves the obstruction and supplies an explicit replacement. The replacement counts physical venue capacity, derives applications and profitable deviations from actual positive-size venues, and optimizes the entire insured-information menu. It also permits positive-cost founder creation and GP entry. The private clue arrives after routing; originators still retire at current closing. These restrictions remain part of the result.

2.1 Why exact-menu market scale matters

Write E for applicant mass and V for funded GP slots. Under $M(E, V) = \chi EV / [(1 + E)(1 + V)]$, a new venue with slot mass ε offers an applicant meeting probability bounded by

$$p_A(E, \varepsilon) = \frac{\chi \varepsilon}{(1 + E)(1 + \varepsilon)} \leq \frac{\chi \varepsilon}{1 + \varepsilon}. \quad (4)$$

Suppose feasible matched incremental founder utility is bounded above by $\bar{a}$. If an alternative route gives a strictly positive incremental value u_0 , no founder applies to the new venue once the right side of (4) times $\bar{a}$ is below u_0 . The conclusion is uniform in its queue and contractual details. A positive application cost can supply the same strict inequality. A positive *absolute* unmatched continuation alone cannot: that continuation cancels when comparing routes.

If positive incremental reservation values hold only almost everywhere, the applicant mass still vanishes as $\varepsilon \downarrow 0$ in a finite population. Its indicator is bounded by $\mathbf{1}_{\{u_x \leq \chi \bar{a} \varepsilon / (1 + \varepsilon)\}}$; dominated convergence applies. The GP meeting probability is then at most χ times that vanishing applicant mass. Avoiding this conclusion with rewards diverging as $1/\varepsilon$ requires actual diverging per-slot collateral, which the bounded productive claims and reserves do not supply.

Consequently a single atomless GP posting a previously unused exact menu can attract no applications even when that menu would be profitable at positive market scale. Positive-size existing venues may therefore support many different menus: the posting deviation is blocked by market creation, not by a proved optimal division of surplus. This does not make every arbitrary menu an equilibrium; existing entry and participation must still hold. It establishes that the continuous empty-market probabilities alone do not prove competitive-search optimality. Under a literal exact-label interpretation, diffuse menu distributions also produce no positive-mass venues and hence no aggregate meetings.

Dividing the same participants equally among n venues gives

$$nM(E/n, V/n) = \frac{\chi n EV}{(n + E)(n + V)}. \quad (5)$$

Its derivative with respect to continuous $n > 0$ has the sign of $EV - n^2$, and its limit as $n \rightarrow \infty$ is zero. For $\chi = 1$ and $E = V = 1/2$, one venue yields $1/9$ matches and two yield $2/25$. Thus menu labels cannot create new copies of the constants 1 without changing the physical technology. Moreover, positive matching rates along shrinking balanced markets are incompatible with a jointly continuous zero extension along the empty axes. A valid architecture must specify which normalization it uses.

2.2 A third physical input and derived application queues

Let a venue have conserved capacity $a > 0$, and apply the original local formula per unit of that capacity:

$$\widehat{M}(E, V, a) = aM(E/a, V/a) = \frac{\chi a EV}{(a + E)(a + V)}. \quad (6)$$

This technology is constant returns in its three physical inputs and satisfies $\widehat{M} \leq \min\{E, V\}$. Dividing all three inputs proportionally preserves total matching. Each potential one-slot GP has $\ell > 0$ nontradable, location-specific venue-capacity units; entry activates them at the GP's stated nonpecuniary cost. Hence $a_m = \ell V_m$, and reposting or splitting a menu does not create capacity. The separate real maintenance input of basic production is not charged again here. Founders' opportunity locations and GP venue locations are fixed; offers compete within each country. Household claims may be held internationally. Cross-country venue relocation and endogenous residence sorting require additional optimization and are not included.

Set $c = \chi\ell/(1 + \ell)$ and $q = E/V$. Substitution gives

$$\widehat{M}(E, V, \ell V) = \frac{cEV}{E + \ell V}, \quad p(q) = \frac{c}{q + \ell}, \quad \lambda(q) = \frac{cq}{q + \ell} = qp(q). \quad (7)$$

The two side probabilities are at most one. The associated common matching measure has dominated applicant and GP marginals, so each founder makes one application and each funded slot obtains at most one venture. Every slot must also reserve its full worst-case financial call; matching capacity is not cash and cannot replace that reservation.

Entry and application precede the privately shared clue. Entry costs are independent of that later information, and all applicants therefore have the same conditional matched prior at a given public state and location. GPs commit their menus before applications. After matching, the actual finite test, publication, consent and insured-settlement game is the finite game specified in Section 1. At each offered feasible menu, sequential follower assessments are admitted; its upper bound covers every such assessment, and the attaining menu is explicitly implemented there. No other financier replaces the committed offer after evidence. Allowing founders to know their clue before routing would permit endogenous type selection and invalidate the scalar reduction below unless that selection were separately solved.

Lemma 2.1 (Actual matched-rent frontier). *Let Q be the household value of the full actual viable productive claim, K its current primary input, and κ the union-certificate probability. Suppose*

$$\bar{\alpha}Q < K < Q, \quad \underline{\alpha}\eta_j(1 - \eta_k)(Q - K) > c_j \quad (j \neq k).$$

Put $W = \kappa(Q - K) - c_F - c_I > 0$. For every feasible continuous menu and consistent participating assessment, founder matched utility r net of its test cost and GP matched cash g satisfy $r + g \leq W$. Every pair $(r, W - r)$ with $0 \leq r \leq W$ is attained by an actual insured menu.

Proof. Consolidate all current transfers, liquidated claims and LP calls. Actual output and LP participation bound the owners' total current net cash by $\mathbb{E}[(\theta Q - K)x]$. Incumbent participation covers its actual test cost. Conditioning on the full private signal profile, a certificate makes financing efficient and joint absence makes it inefficient. The displayed marginal-information inequalities make both tests maximize this surplus at each clue. Subtracting both true test costs gives $r + g \leq W$. This bound covers the complete admitted report, claim, hedge and lottery domain, not only a cash quote or a public no-publication posterior.

To attain it, on histories with both observed tests and required consents pay founder $c_F + r$, incumbent c_I and GP $W - r$, and finance exactly when an authentic certificate is published. On skipped-test and refusal histories disable finance and refund the unused reserve. The founder strictly prefers its reimbursed test when $r > 0$; the incumbent's test is weakly optimal. Select authentic publication, which does not change the fixed receipts, and consent, which preserves them. Matched founder utility is r at each clue. The actual reserve is $R^{LP} = K + \kappa(Q - K)$, and an activated call is $K\mathbf{1}_{\{\text{certificate}\}} + \kappa(Q - K)$. Its expected call is κQ , the value of the actual output claim. Failed-test fees are paid from this earlier advance. Rent division therefore changes neither the total fee nor its existing source. $\square$

Let $u > 0$ be the best gross application utility before deducting creation cost. A used menu with matched rent r must give $rp(q) = u$. Thus routing implies the explicit queue and fill rules

$$q(r, u) = \left(\frac{cr}{u} - \ell \right)_+, \quad \lambda(q(r, u)) = \left(c - \frac{\ell u}{r} \right)_+. \quad (8)$$

If $cr/\ell \leq u$, the offer attracts no applications. At $r = 0$ its queue and fill are zero for $u > 0$.

These formulas have an actual positive-venue foundation. Let μ be a finite measure of existing slots with a positive rent on a positive mass and relevant rents $0 \leq r \leq W$. The source frontier

gives this bound for nonnegative incremental GP rewards; any own-cash offer with $r > W$ has a negative incremental reward and is dominated by no activity and retaining outside cash. This bound ensures a finite first moment and continuity of the integral below; finite slot mass alone would not suffice for unbounded rewards. Let founder supply be $AG(u)$ for a continuous creation-cost CDF G positive at positive arguments. The exact clearing equation is

$$AG(u) = \int q(r, u) \mu(dr). \quad (9)$$

The left side is nondecreasing and bounded; the right side is continuous and strictly decreasing while positive, diverges at zero and vanishes above all useful offers. There is a unique positive root. Adding a new venue of actual size ε and capacity $\ell\varepsilon$ adds $\varepsilon q(r, u)$ to its right side. Strict crossing and continuity imply $u_\varepsilon \rightarrow u$, while its actual applicant mass is $\varepsilon q(r, u_\varepsilon)$ and its GP fill tends to (8). Thus the small-venue deviation is derived from feasible positive markets. It is part of the directed competitive institution, not an arbitrary conditional probability assigned to a null exact label. This differs substantively from (4).

2.3 Global posting and costly participant entry

Theorem 2.2 (Optimal continuous directed offers). *For $0 < u < cW/\ell$, every continuous matched-menu deviation is bounded by*

$$\Pi(r; u) = \left(c - \frac{\ell u}{r}\right)_+ (W - r), \quad 0 \leq r \leq W.$$

The unique maximizing effective founder rent, maximal gross GP entry reward and resulting queue are

$$r^*(u) = \sqrt{\frac{\ell u W}{c}}, \quad \Pi^*(u) = (\sqrt{cW} - \sqrt{\ell u})^2, \quad q^*(u) = \sqrt{\frac{c\ell W}{u}} - \ell. \quad (10)$$

The insured menu of Lemma 2.1 attains the optimum over the entire admitted continuous financial domain.

Proof. Routing gives (8); the full-menu source frontier gives $g \leq W - r$. In the positive-queue region, $\Pi = cW + \ell u - cr - \ell u W/r$. With the displayed r^* ,

$$\Pi^*(u) - \Pi(r; u) = \frac{c(r - r^*)^2}{r} \geq 0. \quad (11)$$

In the no-queue region gross profit is zero, strictly below Π^* . The efficient insured rent split supplies equality at r^* , proving a global deviation result rather than only a first-order condition. $\square$

Let potential founder and GP masses be $A, F > 0$, with continuous CDFs G, J of their nonpecuniary creation and venue-activation costs. Entry costs are paid in utility, not by inventing cash for a zero-cash agent. Suppose $G(0) = J(0) = 0$ and both CDFs are positive at every positive argument. Entry and application clearing reduce to

$$AG(u) = FJ(\Pi^*(u))q^*(u), \quad 0 < u < cW/\ell. \quad (12)$$

The left side is nondecreasing and the right side strictly decreasing while positive. At zero the right side diverges; at the upper endpoint it vanishes while the left side is positive. Thus there is exactly one root, giving actual entry $E = AG(u)$ and $V = FJ(\Pi^*(u))$. Each individual follows its actual cost threshold; the indifferent boundary has zero mass. Different mechanisms attaining the same rent pair need not be unique. Positive lower cost supports are also permitted when the corresponding strict crossing conditions hold.

Corollary 2.3 (Strict entry at bounded population caps). *Let $q_0 = A/F$ and define*

$$u_0 = \frac{c\ell W}{(q_0 + \ell)^2}, \quad r_0 = \frac{\ell W}{q_0 + \ell}, \quad \Pi_0 = \frac{cWq_0^2}{(q_0 + \ell)^2}. \quad (13)$$

If every founder cost is strictly below u_0 and every GP cost strictly below Π_0 , all potential participants strictly enter. Their actual queues, globally optimal menus and entry payoffs are those displayed. With $A = F = 1/2$ and $\chi = \ell = 1$, they reduce to $q_0 = 1$, $r_0 = W/2$, $u_0 = \Pi_0 = W/8$ and matched mass $1/8$. The inequalities may hold in every public state despite state-dependent W , generating the same constant cohort mass from optimizing behavior.

Proof. Substitute q_0 into (10) and the routing indifference equation. The resulting values are (13). The strict cost inequalities make both caps individual best responses; (11) excludes every alternative continuous menu and (9) verifies the applications and new-venue deviation rule. $\square$

3 An exact offer and entry illustration

Take $\ell = \chi = W = 1$, so $c = 1/2$, and $u = 1/8$. Then

$$r^* = 1/2, \quad q^* = 1, \quad \Pi^* = 1/8.$$

The zero-queue boundary is $r = \ell u/c = 1/4$. For $r > 1/4$,

$$\Pi(r; 1/8) = \left(\frac{1}{2} - \frac{1}{8r} \right) (1 - r), \quad \Pi^* - \Pi = \frac{(r - 1/2)^2}{2r}.$$

The illustration normalizes the attainable matched surplus to one. An actual frontier with this value can be obtained, for example, from $\alpha_s = 1/2$, $\eta_F = \eta_I = 1/2$, $c_F = c_I = 1/100$, $Q - K = 68/25$, $K = 3$, $Q = 143/25$. Then $\kappa = 3/8$, $W = 1$, $\bar{\alpha}Q < K < Q$, and each marginal-information benefit is $17/50 > 1/100$. Each slot requires reserve $K + \kappa(Q - K) = 201/50$, separately from its matching capacity.

For a continuous-entry example, set $A = F = 1/2$ and let both creation and activation costs be uniform on $[0, 1/4]$. At $u = \Pi^* = 1/8$, both CDFs equal $1/2$, so $E = V = 1/4$, $q = 1$ and country matched mass $1/16$. The strict crossing argument proves this is the unique positive root; the normalization is a worked conditional example, not an aggregate calibration.

Source and version

This note reproduces the necessary definitions and proofs from the programme's *Optimal information with funded pretest insurance* and *Directed offers with conserved matching capacity*. The focus is the effective offer and entry outcome; distinct financial implementations of the same rent split can coexist. No household-price closure is asserted for this conditional note.