

Innovation, segmentation and state capacity

Nonlinear VCG and Muir: consolidated calibration

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Abstract

This alternative paper rebuilds research incentives, separate country intervention and programme finance on nonlinear Muir and VCG world equilibria. Owned one-period prototype rights, paid installation, deposits and dated refunds determine private prizes; public adopters do not enter private ownership returns. Every mineral has its own equilibrium and each country its own fiscal account. The experiments report periods 0–100 within explicitly finite horizons, fund inherited and terminal liabilities, and retain failures of global attainment, origin backing or numerical consistency. Some finite stock switches require a disclosed atomless selection among globally indifferent cell choices. Infinite-horizon continuation is not certified for this calibration. Comparison with the established dynamic method separates common qualitative conclusions from ownership, technology, institutional and continuation assumptions; the results are not causal estimates.

1 Question, scope and interpretation

The policy question concerns physical extraction, the institutions that transmit a restriction and the financing of technological and public response. A cleaner certificate register alone does not answer all three. This paper supplies an alternative empirical companion built from the latest consolidated theories, with independent model code and the same observed evidence as the extended existing-method papers. It distinguishes an observed scale from a conditional calibration and from a maintained behavioral assumption.

The five mineral markets are separate. Comparable percentage changes and published fixed-reference-value weights provide a global picture without adding incompatible physical tonnes. Main-text comparisons are accompanied by country appendices, source scenarios and a results register retaining failed cases. The agreement of related constructions is not independent empirical confirmation.

2 The five-mineral evidence base

Gold, cobalt, tantalum, tin and tungsten clear in five separate world markets. The reference year is 2024. Coltan denotes an ore or concentrate; the tantalum model uses contained Ta. Gross concentrates, oxide and contained-metal mass are not interchangeable. Prices remain attached to their stated stage: converting an oxide-equivalent quotation into dollars per tonne of metal does not turn it into a measured minegate receipt. This is particularly important for the 3T minerals. (USGS, 2026; BGS, 2026)

The common archive contains 6,277 BGS country–product–year records for 2005–2024 across the five core minerals and copper, nickel and diamonds. Its global country panel is a separately versioned source scenario, not an average of incompatible USGS and BGS observations. Notes identify export proxies, fiscal years and uncertain content. Gross or niobium-containing tantalum

records with no valid Ta conversion remain unconverted. National scales and world residuals are reconciled in the structural registry; the BGS source panel serves the wider descriptive comparison. All observations retain their provenance.

UN Comtrade extraction covers each year from 2005 to 2024, with 13,125 destination margin records, 14,362 origin margin records and 1,021 records in a six-country 3T route panel. These are the declared query populations, not a complete global bilateral network. Missing and zero observations are distinct. The World partner is an aggregate, not an extra country or territory; origin and destination margins must not be added to each other. HS261590 combines niobium, tantalum and vanadium. Without finer national evidence it remains a mixed trade indicator, never pure tantalum extraction or a direct regulatory-exposure measure. HS2615 is broader still and also includes zirconium. (UN Comtrade, 2026; UN Statistics Division, 2017)

IPIS contributes dated, visited mine-site and institutional observations in DRC and CAR. The DRC raw file contains 7,163 records for 3,423 sites; the admitted historical descriptive subset uses onsite IPIS-source visits through 2024. Administrative qualifications and later visits are not silently pooled with that population. Selection into a visit prevents a simple visited-site fraction from identifying the national candidate share. EITI workbooks for 13 country contexts describe fiscal institutions; their often earlier vintages and aggregate currency accounts do not identify 2024 commodity-specific collection. (IPIS, 2026; EITI, 2026)

Rough diamonds provide a certification comparison through the Kimberley Process. Carats, values and certificate records are kept separate: quality and sorting prevent direct insertion into the homogeneous-metal model. Copper and nickel describe industrial and coproduct contexts. Their prices and output do not enter the five-mineral equilibrium unless an explicitly named joint-revenue experiment provides the physical and revenue accounts. Observed 2025 updates remain separate from the 2024 calibration. (Kimberley Process, 2026)

Table 1: Reference scales and fixed-value weights, 2024

Mineral	Mine content (t)	Reference USD/t	Weight (%)
Gold	3,661.2	76,718,111	93.39
Cobalt	302,000.0	26,103	2.62
Tantalum	2,500.0	203,915	0.17
Tin	294,000.0	30,159	2.95
Tungsten	82,000.0	31,778	0.87

Notes: Weights are $P_{m,2024}Q_{m,2024}/\sum_j P_{j,2024}Q_{j,2024}$. Gold retains the original WGC scale; other core minerals use the admitted USGS vintage. Prices concern different product stages. The resulting index is an accounting comparison, not mine revenue or welfare.

3 A mineral world with two physical routes

Each observed raw-mineral unit enters either bulk procurement or a retail processing cell. The two routes are disjoint. Eligible and candidate origin classes are assumed strata, not an illegal/legal classification of whole countries. Candidate and eligible extraction sum to raw-mineral extraction; processed output is a different physical object. Resale reallocates a previously extracted unit and does not add another unit of extraction.

The reference extraction technology is quadratic, $C(q) = kq^2/2$. It implies the supply exponent $\eta = 1$ used for the explicit comparison with the extended empirical method. National production and reference prices fix scale and normalization. Buyer values, retention technology, processing

capacities, route shares and acquisition charges remain assumptions. They cannot be jointly estimated from country production totals.

The Muir cell has regulated mass 0.005, regulated value 2 and resale probability 0.02. Its inverse unregulated demand is piecewise, and its exact revenue envelope is globally optimized together with genuine retention. With primary delivery m , retained conversion y , final-good price p and acquisition charge parameter τ , the solved problem is

$$\max_{m,y \geq 0} p\{V_\tau(m) + H_\delta(y)\} - \frac{k}{2}(m + y)^2.$$

Every stationary point, kink, endpoint and exit in the polynomial pieces is compared. The retained conversion uses the source's H_δ construction, $\delta = .001$ and $h_0 = .2$. The reference charge is .05. Actual final output comes from the allocation and resale integrals, not from dividing sales receipts by price.

The VCG cell has two whole owners, each with one package and independent uniform $[0, 1]$ raw productivity. Acquisition charges enter the affected owner's net value. The pivot payment removes the entire owner, not an artificial marginal fraction of that owner. Seller retention is genuine, with value phq , $h = .1$ and $k = .2$. Expected payments, retained stock and physical conversion are evaluated by exact polynomial integrals.

The baseline commits all extracted inventory to the specified mechanism. Making a second indivisible package available can change the receipt schedule discontinuously. The solver compares inventory branches and checks attainment. A supremum approached just below a package boundary is not a valid maximizer. Endogenous withholding is a separately named alternative, not an automatic repair to the all-offered model.

4 Origin claims and three simultaneous clearing equations

The bulk contract retains the canonical funded specification: initial late discovery .25, policy discovery .50, deposit .20, forgery curvature 2, diligence curvature .50, early/late rewards .02 and 1, and fee .02. Fraud and diligence are endogenous; total detection combines policy-set late discovery with endogenous diligence within the declared domains. No-silence, supplier participation and origin backing are checked explicitly.

Let Q_v be vulnerable eligible bulk output, π the fraud fraction and d_π the total detection probability. Diverted candidate mass is $D_v = Q_v\pi/(1 - \pi)$; certificate entries are $X = Q_C^{bulk} + D_v$; outside delivery is $Y = Q_D^{bulk} - D_v$. Revoked claims are $d_\pi D_v$ and live candidate claims are $(1 - d_\pi)D_v$. A negative outside-delivery stock cannot be rescued by aggregate price clearing.

Let h be the eligible bulk quote, p_D the candidate bulk quote, p the processed-good quote, f the fee and ℓ the expected buyer loss on a vulnerable entry. With actual processed output Z_R the equations are

$$h\{(1 + f)X + \ell Q_v/(1 - \pi)\} = E_C, \tag{1}$$

$$p_D Y = E_D + \epsilon \frac{Y}{Y + Z_R}, \tag{2}$$

$$p Z_R = E_R + \epsilon \frac{Z_R}{Y + Z_R}. \tag{3}$$

The assumed small coupling is $\epsilon = .001 \min(E_{D0}, E_{R0})$. Reference expenditures fit the baseline at $(h, p_D, p) = (1, .8, 1)$. This exact conditional fit does not identify a demand system. The raw-

mineral world normalizer scales country extraction once; Z_R remains a distinct processed-product unit, not an additional tonne of mineral.

The default static route fraction is .25 for eligible and candidate sources, with .10 and .50 alternatives. The bulk-only compatibility branch removes retail and coupling and separately solves the public-price equations. It is a diagnostic at a different boundary, not an application of the positive-core theorem where its hypotheses no longer hold.

5 Owned research and public adoption

The dynamic economy expands independently licensed installations rather than changing the extraction coefficient inside a cell. Each clean or candidate private core owns the incremental operating return from its own prototype. Public adopters are excluded from that owner's prize. Candidate research owns the unreformed, nonreachable private core; ROW is a fixed private core without a reforming government.

The source's short-prize subclass uses one-period prototype/access leases, no salvage and public replacement access that is identical after disclosure across an individual's success and failure histories. An owner is negligible and takes next-date prices as given; aggregate successes reprice the whole world. The fitted improvement scalar uses an assumed R&D/revenue ratio .005. This one moment does not identify the success technology or ownership regime.

Science supply is .8, success is $1 - \exp[-\log(2)s]$, laboratory curvature is .002 and the discount factor is $\beta = .95$. A new portfolio pays .001 in lease/installation costs per normalized physical unit. The clean portfolio also posts its own incremental safe-bulk bond, .20 of its invoice, refundable after three periods. If $\Delta\Pi_j$ is the next-date owned operating increment, I_j the paid lease and D_C the clean bond, its prize is

$$J_C = \beta(\Delta\Pi_C - I_C - D_C) + \beta^4 D_C, \quad (4)$$

$$J_D = \beta(\Delta\Pi_D - I_D). \quad (5)$$

The refund has its own date. An extra carrying charge is not deducted again from a prize already discounting deposit and refund. Private residual rights are zero under the stated lease. Later service payments are zero only because a separately funded trust prepays them; its resource and financial cost remains in the programme accounts.

Private science satisfies both research first-order/complementarity conditions, scientific resource feasibility and complementary slackness. Supported science directs .8 to clean research with the smallest nonnegative success-contingent reward that deters dirty research, plus a disclosed 10^{-11} numerical tie-break. Grants are committed now and paid next date. Frozen research is an imposed experiment, not a private optimum.

6 Country governments and the timing of reform

Each government compares the continuation value of protection against an attempt to extend legal capacity. Taxes apply to actual optimized legal operating profit. The reference protection payment is .40 of candidate unit operating profit, success probability .25, reachable fraction .25, attempt cost .25 and legal service cost .01. These are scenario primitives. Nominal tax exposure, effective collection and productive reach are distinct.

The backward recursion implements the consolidated source’s Bellman comparison. Intervention gives up current protection, and successful adoption first operates next date. Country masses evolve consistently with the selected actions. Countries with identical assumed decision primitives may share a computational type, but their scales, masses and fiscal budgets remain separate. Colombia, Peru and DRC retain their benchmark tax assumptions; .30 in other countries is a labelled assumption. ROW never acquires a government or a public budget.

The aggregate price path and every country’s action must be jointly consistent. Fraud-organizer income belongs to its specified organizer, not automatically to government protection revenue. A positive world tax total therefore cannot establish the incentive or financing of a particular country’s intervention.

7 Finite experiments and continuation discipline

The principal numerical experiment uses $T = 120$, with $T = 140$ horizon checks, and reports periods 0–100. Date 101 is retained to evaluate date 100 incentives. There are no new production, research or intervention commitments after T ; existing bond maturities and permanent records remain fully funded. No model period is converted into a calendar year.

The supported-growth reference uses eligible retail routing .25 and candidate routing .0025, a separately versioned small-core scenario. The equal-routing static reference remains a distinct stress. This routing change is not an estimated response or a hidden recalibration. Private and frozen research with growing demand, and explicit fixed-expenditure alternatives, are each retained as their own cases. A valid fixed-expenditure path does not replace a failed growth path in a paired effect. The normalized intervention-cost scale still grows at the fitted clean-support reference rate in both demand scenarios; “fixed expenditure” holds consumer expenditures fixed, not that cost scale.

Leaving the printed price neighborhood $[.99, 1.01]$ is recorded separately from failure of a globally optimized finite solution. At a genuine cell indifference, an explicitly labelled atomless mixture can split cells between two globally optimal stocks. Each VCG cell retains whole owners and all-offered inventory. Indifference, feasible mixing and physical clearing are checked independently. This finite equilibrium selection lies outside the source’s strict unique-branch theorem. A nonattained supremum cannot be mixed into a solution. Infinite-horizon continuation remains NOT_CERTIFIED. The algorithm verifies each admitted finite path’s equations and optimality; it does not enumerate every government fixed point or prove that policy effects are invariant to equilibrium selection. Paired results concern the selected finite paths. This possible government-path multiplicity is distinct from the explicitly checked cell-indifference mixture.

Table 2: Recorded nonlinear dynamic cases

Status	Cases
solved conditional finite horizon	401
unresolved or unlicensed	189

Notes: The coverage ledger retains failures and the reason for each. These counts describe the executed design, not statistically independent observations.

The central supported design admits 29 of its 30 paths. Tungsten–VCG under combined policy remains unresolved. Gold and cobalt retain small screening responses. The central 3T screening changes at period 100 are numerical zero: both selected paths have nearly exhausted the reachable unreformed population and the bulk fraud margin is inactive. This late-period

Table 3: Declared dynamic coverage by research regime

Research	Mechanism	Planned	Recorded	Admitted	Excluded
frozen	Muir	95	95	85	10
frozen	VCG	95	95	55	40
private	Muir	110	110	60	50
private	VCG	110	110	76	34
supported	Muir	90	90	61	29
supported	VCG	90	90	64	26

Notes: Includes both expenditure paths, government and market configurations, sensitivities and horizon checks. Exclusions retain their specific reasons in the ledger; a numerical failure is not proof of nonexistence.

Table 4: Combined-policy candidate extraction across research regimes

Research	Mechanism	Paired minerals	Minimum (%)	Maximum (%)
frozen	Muir	1/5	-13.2883	-13.2883
frozen	VCG	0/5	–	–
private	Muir	2/5	-16.8706	-12.7333
private	VCG	2/5	-17.5554	-12.7306
supported	Muir	5/5	-13.3532	-12.7333
supported	VCG	4/5	-13.3534	-12.7306

Notes: Period-100 changes from own baseline across admitted mineral pairs only: segmented markets, active governments, T120, growth demand, central institutional primitives and small candidate retail core. Coverage differs; these ranges are neither confidence intervals nor an isolated causal research effect.

Table 5: First country intervention attempts in admitted supported pairs

Mineral	Mechanism	Policy	First-date range	Countries
Gold	Muir	baseline	15–26	2
Gold	Muir	combined	7–14	2
Cobalt	Muir	baseline	27	12
Cobalt	Muir	combined	15	12
Tantalum	Muir	baseline	20	11
Tantalum	Muir	combined	12	11
Tin	Muir	baseline	20–36	14
Tin	Muir	combined	11–25	14
Tungsten	Muir	baseline	20	10
Tungsten	Muir	combined	11	10
Gold	VCG	baseline	21–32	2
Gold	VCG	combined	11–20	2
Cobalt	VCG	baseline	31	12
Cobalt	VCG	combined	20	12
Tantalum	VCG	baseline	24	11
Tantalum	VCG	combined	15	11
Tin	VCG	baseline	24–40	14
Tin	VCG	combined	15–30	14

Notes: For each country, the first intervention action above the numerical reporting threshold of 0.00001 through period 100 is counted, whether or not it succeeds. The range is across those countries. None means no action above that threshold within the reported window; excluded pairs are omitted rather than assigned zero. No date is a calendar year.

Table 6: Joint state-capacity profiles: admitted combined-policy pairs

Profile	Mechanism	Paired minerals	Min. (%)	Max. (%)	First attempt
central	Muir	5/5	-13.3532	-12.7333	7–15
central	VCG	4/5	-13.3534	-12.7306	11–20
low capture	Muir	2/5	-13.7226	-13.4738	50–72
low capture	VCG	2/5	-13.3611	-12.7413	57–57
enabling	Muir	1/5	-13.3533	-13.3533	4–4
enabling	VCG	2/5	-13.3534	-12.7296	7–14

Notes: Period-100 candidate extraction change from each profile’s own baseline. Low capture changes collection to .5. Enabling changes success to .40 and reach to .30. Reach also changes ownership; the common .005 research/revenue fit target is refitted, so derived growth, demand growth and cost normalization can change across profiles. These are conditional packages, not isolated primitive effects. First attempt spans mineral worlds with a country action above the numerical reporting threshold of 0.00001 through 100; none means no action above that threshold within the window.

Table 7: Supported-growth comparison at period 100

Mineral	Mechanism	Policy	Candidate (%)	s_C	s_D
Gold	Muir	screening	-0.0067	0.80	0.00
Gold	Muir	combined	-13.3532	0.80	0.00
Gold	VCG	screening	-0.0067	0.80	0.00
Gold	VCG	combined	-13.3534	0.80	0.00
Cobalt	Muir	screening	-0.4008	0.80	0.00
Cobalt	Muir	combined	-12.7333	0.80	0.00
Cobalt	VCG	screening	-0.4023	0.80	0.00
Cobalt	VCG	combined	-12.7306	0.80	0.00
Tantalum	Muir	screening	-0.0000	0.80	0.00
Tantalum	Muir	combined	-13.2770	0.80	0.00
Tantalum	VCG	screening	-0.0000	0.80	0.00
Tantalum	VCG	combined	-13.2728	0.80	0.00
Tin	Muir	screening	-0.0000	0.80	0.00
Tin	Muir	combined	-13.2763	0.80	0.00
Tin	VCG	screening	0.0000	0.80	0.00
Tin	VCG	combined	-13.2720	0.80	0.00
Tungsten	Muir	screening	0.0000	0.80	0.00
Tungsten	Muir	combined	-13.2701	0.80	0.00
Tungsten	VCG	screening	-0.0000	0.80	0.00
Tungsten	VCG	combined	–	–	–

Notes: Separate country governments, segmented markets, T120, small candidate retail core. Changes are relative to the same mechanism’s own baseline. A dash means an admitted matched pair is unavailable, not a zero effect.

comparison is not a claim that stronger discovery had no earlier effect or that all institutions are equivalent. Among admitted central pairs, the combined policy reduces candidate extraction by about 12.7–13.4%; it also imposes the declared outside-spending contraction.

The separate T120/T140 diagnostic contains 76 admitted matched paths and 4,560 date/outcome comparisons. The largest raw-quantity difference is 6.52×10^{-8} in normalized world units; the largest technology difference is 2.32×10^{-7} . Only the observed matched paths and dates receive this stability statement. Failed horizon pairs remain unavailable, and this finite check does not establish infinite continuation.

Table 8: Central cash bridge through period 100: supported-growth scenario

Mineral	Mechanism	Baseline / reference value	Combined / reference value
Gold	Muir	228.037	228.007
Gold	VCG	48.103	48.043
Cobalt	Muir	309.598	309.899
Cobalt	VCG	69.708	69.712
Tantalum	Muir	306.395	306.994
Tantalum	VCG	63.963	63.932
Tin	Muir	306.058	306.663
Tin	VCG	63.894	63.863
Tungsten	Muir	303.509	304.087
Tungsten	VCG	63.348	–

Notes: Maximum cumulative central deficit divided by that mineral’s full 2024 reference value, not a percentage. Large ratios reflect assumed custody carrying payments, service funding and grants. Initial inherited reserves and custody-provider capitalization are reported separately; this flow bridge excludes them. These are conditional requirements, not estimated programme budgets or evidence of affordability. Each policy has its own full cash ledger.

Table 9: Separate actor accounts: supported-growth combined policy

Mineral	Mechanism	Inherited deposits	Service reserve	Custody capital	Private cash need
Gold	Muir	0.229	0.00043	12.523	9.266
Gold	VCG	0.229	0.00043	2.568	10.094
Cobalt	Muir	0.362	0.00068	19.541	11.563
Cobalt	VCG	0.362	0.00068	4.007	14.367
Tantalum	Muir	0.227	0.00042	12.315	13.325
Tantalum	VCG	0.227	0.00042	2.525	15.310
Tin	Muir	0.227	0.00042	12.315	13.308
Tin	VCG	0.227	0.00042	2.525	15.287
Tungsten	Muir	0.227	0.00042	12.315	13.166
Tungsten	VCG	–	–	–	–

Notes: All entries are multiples of the mineral’s full 2024 reference value. The first three columns are distinct initial stocks, not treasury fee revenue. Private cash need covers the full finite horizon 0–120, including dated installation payments, before operating sales receipts; it is not net profit or a private-sector funding gap. The preceding central bridge reports 0–100. Do not sum these columns into a programme budget.

8 Deposits, service trusts and actor-specific funding

Three inherited certification cohorts and their permanent records predate period 0 under all counterfactuals. New deposits, refunds and expected forfeitures are tracked by original vintage. Principal is never spent as fee revenue. Safe-core research bonds have no false-origin forfeiture; forfeiture transfers have no presumed treasury beneficiary.

Permanent record service costs .00001 per registered unit per period and active claims .0001 through maturity. A reserve prepays each cohort’s full discounted service liability. It earns an assumed gross safe return 1/.95. Both the reserve’s dated asset recurrence and the remaining liability value are independently reconstructed. These funding and interest-rate assumptions do not establish the feasibility of an actual programme.

Retail custody uses separate providers and funded collectible capital. The reference pool is 20p per cell, assumed unsuppressible discovery .25 and bounded coalition gain 2.1p. The

positive deterrence margin applies only under those institutional restrictions, including enforceable knowing vetoes. Provider capital is distinct from producer wealth. Its .05 carrying payment and .001 service cost per extracted retail unit are paid by the nonmineral treasury and are shown as required appropriations. They do not quietly lower an owner's already-net prize. Retail allocations, resale and retained conversion close within each date; the provider pool is returnable after the last transaction. No separate post-terminal retail custody-service stream is assumed. Permanent records and remaining active claims belong to the distinct bulk certification cohorts.

The saved accounts distinguish fee receipts, central top-ups, grant commitments and payments, dated private installation commitments and payments, principal, service reserves, custody capital and country budgets. Private installation payments go to the installation/access provider in the next period after research success. The private programme liquidity measure includes these payments and is reported before operating sales receipts; it is not net profit or a full wealth account. The treasury separately funds custody, services and grants. Country financing bridges are computed from each country's own cash history. An arithmetic total across them does not authorize international transfers. The public country budget comprises captured legal tax less legal service and attempt costs in newly reformed reachable territory. It excludes preexisting national revenues. Protection income affects the government's intervention value but is not booked as public tax revenue; the bridge is a programme financing requirement, not a complete account of political or private protection income. The reported central bridge is the maximum cumulative net treasury flow deficit, without interest on treasury balances. It excludes the initial inherited reserve endowment and provider capitalization, which are shown separately. Present-valued positive top-ups are a different appropriation measure; only the dedicated service reserve earns the stated safe return. At the terminal date, refundable stocks keep their original maturity and permanent records retain their full funded liability.

9 What the alternative reconstruction establishes

The model makes ownership, timing, retention and finance explicit enough to test the proposed mechanisms. It does not estimate buyer values, collection rates, research success or a causal certification effect from aggregate mineral data. The older dynamic calibration and this rebuilt economy can agree about a leakage ratio while differing about owner profits, technological growth, adoption timing and funding. Such differences must be attributed to the corresponding assumptions before drawing a policy conclusion.

Unresolved or inadmissible cases are substantive limits of the delivered calibration, not negligible missing observations. In particular, a finite successful example is not a proof of the infinite-horizon result. The evidence archive, executable specification, full coverage ledger and independent reconstruction make those limits reviewable. Failure of the implemented pure or selected-mixture branches to clear a market is not a proof that every possible equilibrium is nonexistent.

Table 10: Candidate extraction under stronger discovery: matched curvature

Mineral	Existing method, $\eta = 1$	Nonlinear Muir	Nonlinear VCG
Gold	-0.00888	-0.00666	-0.00666
Cobalt	-0.50559	-0.37916	-0.37915
Tantalum	-0.13541	-0.10155	-0.10155
Tin	-0.13504	-0.10128	-0.10128
Tungsten	-0.13207	-0.09905	-0.09905

Notes: Percentage changes from each model’s own baseline, outside spending fixed. B sends 25% of both source classes to retail; A has the original bulk construction. Unknown country shares are 0.5. These rows match curvature and observed scales, not every institutional primitive.

A Executable source crosswalk and independent reconstruction

Both consolidated theoretical parents are frozen beside this edition, with their hashes and equation labels. The static implementation maps the source objects `nw:cellproblem`, `nw:V`, `nw:retention`, `nw:actualfinal`, `nw:vcgcontinuation`, `nw:retailclearing` and `nw:jointclearing` into exact cell and world accounts. Dynamic ownership and government timing follow the specified subclasses of `dg:prize`, `dg:bellman` and `dg:population`. The readable implementation specification records every numerical primitive, domain, fitted moment and additional finite-equilibrium selection.

The independent static review reconstructs 475 saved states, 36 separate Muir global optimizations and direct whole-owner allocation/removal calculations using 1.44 million buyer-value pairs per tested VCG case. It also compares 150 bulk-only states with the original method at $\eta = 1$. Its 13,695 checks pass without modifying the equilibrium engine. Verification of a numerical example does not estimate its buyer primitives or validate an unrestricted theorem beyond its hypotheses.

The Muir reference extraction, final output and profit are approximately 0.2050386143, 0.1527677063 and 0.0984661155. The VCG reference extracts one package, obtains expected receipts .343 and profit .243, produces .667 physical final units and retains .01 expected mineral units. Receipt units and final-product units are visibly distinct even at these reference points.

For a VCG retention-only/one-package global switch the lower positive price root is

$$p = \frac{k}{h^2} \{R_h - \sqrt{R_h^2 - h^2}\},$$

with competing stocks ph/k and 1. A finite mixture checks equality of their global profits and an aggregate fraction in $[0, 1]$. It does not split an individual owner. Other nonconvex branches retain exact global comparisons and explicit nonattainment diagnostics.

Dynamic verification reads saved paths without importing the solver. It reconstructs physical markets, origins, owner leases and deposit/refund prizes, research conditions, country Bellman choices and transitions, inherited liabilities, grants and the permanent service reserve. Static/research admission is 10^{-8} and government admission 10^{-6} , with distinct grid and horizon checks. All excluded cases remain in the coverage denominator.

B Country scales, assumptions and missing calibration

The following registry is shared by the structural exercises. “Range” means the assumed candidate fractions 0.1, 0.5 and 0.9, not measured illegal or artisanal shares. Existing gold and DRC cobalt fractions retain their original transport qualifications. ROW is the source residual with a fixed 0.5 candidate split, no vulnerable certificate origin and no government. Uganda tantalum and DRC/Burundi/Uganda tungsten remain descriptive BGS cases without a named structural scale. Zero production and missing calibration are distinct. The fuller BGS producer tabulation is supplied as a separate CSV source scenario; its country coverage should not be confused with the narrower gold calibration.

Table 11: Admitted structural production registry

Mineral	Country or residual	Tonnes	Candidate share
Gold	Peru	108.15	0.1873 (transported)
Gold	Colombia	56.70	0.5386 (transported)
Gold	Rest of world and source round- ing balance	3,496.35	0.5 (assumed resid- ual)
Cobalt	United States	200.00	Range (assumed)
Cobalt	Australia	4,780.00	Range (assumed)
Cobalt	Canada	3,350.00	Range (assumed)
Cobalt	China	2,000.00	Range (assumed)
Cobalt	Democratic Republic of the Congo	226,000.00	0.1000 (transported)
Cobalt	Cuba	3,450.00	Range (assumed)
Cobalt	Indonesia	35,000.00	Range (assumed)
Cobalt	Madagascar	3,100.00	Range (assumed)
Cobalt	Papua New Guinea	2,630.00	Range (assumed)
Cobalt	Philippines	3,100.00	Range (assumed)
Cobalt	Russia	8,000.00	Range (assumed)
Cobalt	Turkey	2,200.00	Range (assumed)
Cobalt	Rest of world and source round- ing balance	8,190.00	0.5 (assumed resid- ual)
Tantalum	United States	0.00	Zero scale; no experi- ment
Tantalum	Australia	52.00	Range (assumed)
Tantalum	Bolivia	2.00	Range (assumed)
Tantalum	Brazil	210.00	Range (assumed)
Tantalum	Burundi	2.00	Range (assumed)
Tantalum	China	76.00	Range (assumed)
Tantalum	Democratic Republic of the Congo	1,270.00	Range (assumed)
Tantalum	Ethiopia	40.00	Range (assumed)
Tantalum	Mozambique	55.00	Range (assumed)
Tantalum	Nigeria	390.00	Range (assumed)
Tantalum	Russia	29.00	Range (assumed)
Tantalum	Rwanda	374.00	Range (assumed)

Mineral	Country or residual	Tonnes	Candidate share
Tin	United States	0.00	Zero scale; no experiment
Tin	Australia	11,300.00	Range (assumed)
Tin	Bolivia	21,200.00	Range (assumed)
Tin	Brazil	27,600.00	Range (assumed)
Tin	Myanmar	20,000.00	Range (assumed)
Tin	China	71,000.00	Range (assumed)
Tin	Democratic Republic of the Congo	26,000.00	Range (assumed)
Tin	Indonesia	55,000.00	Range (assumed)
Tin	Laos	1,860.00	Range (assumed)
Tin	Malaysia	5,460.00	Range (assumed)
Tin	Nigeria	3,100.00	Range (assumed)
Tin	Peru	32,300.00	Range (assumed)
Tin	Russia	3,260.00	Range (assumed)
Tin	Rwanda	4,100.00	Range (assumed)
Tin	Vietnam	11,000.00	Range (assumed)
Tin	Rest of world and source rounding balance	820.00	0.5 (assumed residual)
Tungsten	United States	0.00	Zero scale; no experiment
Tungsten	Australia	920.00	Range (assumed)
Tungsten	Austria	840.00	Range (assumed)
Tungsten	Bolivia	1,700.00	Range (assumed)
Tungsten	China	67,000.00	Range (assumed)
Tungsten	Kazakhstan	0.00	Zero scale; no experiment
Tungsten	North Korea	1,900.00	Range (assumed)
Tungsten	Portugal	650.00	Range (assumed)
Tungsten	Russia	1,500.00	Range (assumed)
Tungsten	Rwanda	1,300.00	Range (assumed)
Tungsten	Spain	700.00	Range (assumed)
Tungsten	Vietnam	3,400.00	Range (assumed)
Tungsten	Rest of world and source rounding balance	2,090.00	0.5 (assumed residual)

C Separate country programme financing

Each entry is the country's own maximum cumulative public cash deficit through period 100 in the supported-growth, segmented, T120, small-retail-core scenario. The values use the source-stage 2024 reference-price scale and are millions of USD. The full paired states, source shares and actor accounts remain available. These are public programme budgets for the reachable/reforming model populations, excluding preexisting national revenues and political/private protection income. No offset across countries or transfer arrangement is assumed. A dash denotes an unavailable admitted path.

Table 12: Country cash bridge, reference USD million

Mineral	Mechanism	Country	Baseline	Combined
Gold	Muir	Peru	360.7495	277.3411
Gold	Muir	Colombia	398.0582	334.1332
Gold	VCG	Peru	445.0854	337.8433
Gold	VCG	Colombia	490.5960	389.9165
Cobalt	Muir	United States	0.3849	0.3017
Cobalt	Muir	Australia	9.1987	7.2111
Cobalt	Muir	Canada	6.4468	5.0538
Cobalt	Muir	China	3.8488	3.0172
Cobalt	Muir	Democratic Republic of the Congo	86.9840	68.1889
Cobalt	Muir	Cuba	6.6393	5.2047
Cobalt	Muir	Indonesia	67.3548	52.8012
Cobalt	Muir	Madagascar	5.9657	4.6767
Cobalt	Muir	Papua New Guinea	5.0612	3.9676
Cobalt	Muir	Philippines	5.9657	4.6767
Cobalt	Muir	Russia	15.3954	12.0688
Cobalt	Muir	Turkey	4.2337	3.3189
Cobalt	VCG	United States	0.4888	0.4029
Cobalt	VCG	Australia	11.6813	9.6287
Cobalt	VCG	Canada	8.1867	6.7481
Cobalt	VCG	China	4.8876	4.0287
Cobalt	VCG	Democratic Republic of the Congo	110.4595	91.0496
Cobalt	VCG	Cuba	8.4311	6.9496
Cobalt	VCG	Indonesia	85.5328	70.5030
Cobalt	VCG	Madagascar	7.5758	6.2445
Cobalt	VCG	Papua New Guinea	6.4272	5.2978
Cobalt	VCG	Philippines	7.5758	6.2445
Cobalt	VCG	Russia	19.5504	16.1150
Cobalt	VCG	Turkey	5.3763	4.4316
Tantalum	Muir	Australia	0.6802	0.5448
Tantalum	Muir	Bolivia	0.0262	0.0210
Tantalum	Muir	Brazil	2.7471	2.2002
Tantalum	Muir	Burundi	0.0262	0.0210
Tantalum	Muir	China	0.9942	0.7963
Tantalum	Muir	Democratic Republic of the Congo	16.6137	13.3062
Tantalum	Muir	Ethiopia	0.5233	0.4191

Mineral	Mechanism	Country	Baseline	Combined
Tantalum	Muir	Mozambique	0.7195	0.5763
Tantalum	Muir	Nigeria	5.1018	4.0862
Tantalum	Muir	Russia	0.3794	0.3038
Tantalum	Muir	Rwanda	4.8925	3.9185
Tantalum	VCG	Australia	0.8236	0.6603
Tantalum	VCG	Bolivia	0.0317	0.0254
Tantalum	VCG	Brazil	3.3260	2.6668
Tantalum	VCG	Burundi	0.0317	0.0254
Tantalum	VCG	China	1.2037	0.9651
Tantalum	VCG	Democratic Republic of the Congo	20.1147	16.1277
Tantalum	VCG	Ethiopia	0.6335	0.5080
Tantalum	VCG	Mozambique	0.8711	0.6984
Tantalum	VCG	Nigeria	6.1769	4.9526
Tantalum	VCG	Russia	0.4593	0.3683
Tantalum	VCG	Rwanda	5.9235	4.7494
Tin	Muir	Australia	23.1432	20.4226
Tin	Muir	Bolivia	43.4190	38.3149
Tin	Muir	Brazil	56.5267	49.8817
Tin	Muir	Myanmar	40.9614	36.1461
Tin	Muir	China	145.4128	128.3188
Tin	Muir	Democratic Republic of the Congo	53.2498	46.9900
Tin	Muir	Indonesia	112.6437	99.4019
Tin	Muir	Laos	3.8094	3.3616
Tin	Muir	Malaysia	11.1825	9.8679
Tin	Muir	Nigeria	6.3490	5.6027
Tin	Muir	Peru	170.7410	128.9931
Tin	Muir	Russia	6.6767	5.8918
Tin	Muir	Rwanda	8.3971	7.4100
Tin	Muir	Vietnam	22.5287	19.8804
Tin	VCG	Australia	29.0175	23.8802
Tin	VCG	Bolivia	54.4399	44.8017
Tin	VCG	Brazil	70.8745	58.3268
Tin	VCG	Myanmar	51.3584	42.2658
Tin	VCG	China	182.3222	150.0435
Tin	VCG	Democratic Republic of the Congo	66.7659	54.9455
Tin	VCG	Indonesia	141.2355	116.2309
Tin	VCG	Laos	4.7763	3.9307
Tin	VCG	Malaysia	14.0208	11.5386
Tin	VCG	Nigeria	7.9605	6.5512
Tin	VCG	Peru	214.2297	163.4251
Tin	VCG	Russia	8.3714	6.8893
Tin	VCG	Rwanda	10.5285	8.6645
Tin	VCG	Vietnam	28.2471	23.2462
Tungsten	Muir	Australia	1.8160	1.6890
Tungsten	Muir	Austria	1.6581	1.5421

Mineral	Mechanism	Country	Baseline	Combined
Tungsten	Muir	Bolivia	3.3557	3.1210
Tungsten	Muir	China	132.2541	123.0044
Tungsten	Muir	North Korea	3.7505	3.4882
Tungsten	Muir	Portugal	1.2831	1.1933
Tungsten	Muir	Russia	2.9609	2.7538
Tungsten	Muir	Rwanda	2.5661	2.3867
Tungsten	Muir	Spain	1.3818	1.2851
Tungsten	Muir	Vietnam	6.7114	6.2420
Tungsten	VCG	Australia	2.2158	—
Tungsten	VCG	Austria	2.0231	—
Tungsten	VCG	Bolivia	4.0945	—
Tungsten	VCG	China	161.3702	—
Tungsten	VCG	North Korea	4.5762	—
Tungsten	VCG	Portugal	1.5655	—
Tungsten	VCG	Russia	3.6128	—
Tungsten	VCG	Rwanda	3.1311	—
Tungsten	VCG	Spain	1.6860	—
Tungsten	VCG	Vietnam	8.1889	—

D Measurement and procurement admission

The oxide conversion for tantalum uses the declared Ta fraction 0.81897 in Ta_2O_5 . A quotation of USD 167 per kg oxide-equivalent therefore gives $167000/0.81897$ USD per tonne of contained Ta. Tungsten uses the declared 0.793 W fraction in WO_3 : one dry metric tonne unit is 10 kg WO_3 , or 7.93 kg W, giving $252000/7.93$ USD/t W for a USD 252/dmtu reference. Cobalt and tin pound quotations use 0.45359237 kg/lb. All these conversions are checked separately from their suitability as procurement prices.

The Rwanda 3T study provides evidence of assays, payment stages and food deductions, but its tantalum table is not admitted as an elasticity moment. The COMIKAGI table reports 1,540 kg concentrate per month at 30% Ta_2O_5 , a USD 130/kg quotation of unclear basis, and USD 37.7/kg concentrate locally. The displayed international total USD 200,200 treats the quotation as per kg concentrate; an oxide-content application instead gives USD 60,060. The table's combined total USD 514,578 also differs from USD 45,619 plus USD 200,200. The local total USD 58,058 is internally consistent. These alternatives imply radically different ratios, so no tantalum procurement elasticity is inferred. ([Rwanda study, 2018](#))

The cassiterite example is arithmetically coherent: 3,960 kg concentrate, 60% tin and USD 19.20/kg metal imply approximately USD 45,619 internationally; USD 11/kg concentrate implies USD 43,560 locally. The ratio is approximately 0.955. Gifurwe's tungsten comparison gives approximately 0.686 from USD 8.4/kg concentrate and USD 188.5/dmtu at 65% WO_3 . Both are transaction-stage ratios, not supply elasticities. The independently inspected pages and all arithmetic are retained in the Rwanda procurement audit.

Observed scales, conditional fits and assumptions have distinct records. No new mineral inherits the Colombia or Peru procurement elasticity merely because it is artisanal or traded

through the same region. Identifying that elasticity requires matched transaction prices, assay and deduction data, production costs, net margins and a defensible population bridge.

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