

Transmission and institutional resilience

Nonlinear VCG and Muir: consolidated calibration

Elies Lucio

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Abstract

This alternative paper rebuilds the institutional-resilience exercise using nonlinear Muir selling and whole-owner VCG payments with genuine retention. The household procurement model is preserved, but each selling institution and fixed-route comparator is stated separately. Eight paired configurations produce 216 finite allocations and 18 Shapley decompositions, accompanied by a grid refinement. Global trade, mine-site and fiscal evidence identifies relevant measurement questions without being mistaken for effective regulatory exposure or a fitted household population. The comparison with the existing-method extension isolates the interpretation of retention, buyer removal, resale and outside options. Mineral repetitions are controlled hypothetical applications, not independent empirical replications. Unsupported country parameters remain missing or explicit scenario assumptions.

1 Question, scope and interpretation

The policy question concerns physical extraction, the institutions that transmit a restriction and the financing of technological and public response. A cleaner certificate register alone does not answer all three. This paper supplies an alternative empirical companion built from the latest consolidated theories, with independent model code and the same observed evidence as the extended existing-method papers. It distinguishes an observed scale from a conditional calibration and from a maintained behavioral assumption.

The five mineral markets are separate. Comparable percentage changes and published fixed-reference-value weights provide a global picture without adding incompatible physical tonnes. Main-text comparisons are accompanied by country appendices, source scenarios and a results register retaining failed cases. The agreement of related constructions is not independent empirical confirmation.

2 The five-mineral evidence base

Gold, cobalt, tantalum, tin and tungsten clear in five separate world markets. The reference year is 2024. Coltan denotes an ore or concentrate; the tantalum model uses contained Ta. Gross concentrates, oxide and contained-metal mass are not interchangeable. Prices remain attached to their stated stage: converting an oxide-equivalent quotation into dollars per tonne of metal does not turn it into a measured minegate receipt. This is particularly important for the 3T minerals. (USGS, 2026; BGS, 2026)

The common archive contains 6,277 BGS country–product–year records for 2005–2024 across the five core minerals and copper, nickel and diamonds. Its global country panel is a separately versioned source scenario, not an average of incompatible USGS and BGS observations. Notes identify export proxies, fiscal years and uncertain content. Gross or niobium-containing tantalum records with no valid Ta conversion remain unconverted. National scales and world residuals

are reconciled in the structural registry; the BGS source panel serves the wider descriptive comparison. All observations retain their provenance.

UN Comtrade extraction covers each year from 2005 to 2024, with 13,125 destination margin records, 14,362 origin margin records and 1,021 records in a six-country 3T route panel. These are the declared query populations, not a complete global bilateral network. Missing and zero observations are distinct. The World partner is an aggregate, not an extra country or territory; origin and destination margins must not be added to each other. HS261590 combines niobium, tantalum and vanadium. Without finer national evidence it remains a mixed trade indicator, never pure tantalum extraction or a direct regulatory-exposure measure. HS2615 is broader still and also includes zirconium. (UN Comtrade, 2026; UN Statistics Division, 2017)

IPIS contributes dated, visited mine-site and institutional observations in DRC and CAR. The DRC raw file contains 7,163 records for 3,423 sites; the admitted historical descriptive subset uses onsite IPIS-source visits through 2024. Administrative qualifications and later visits are not silently pooled with that population. Selection into a visit prevents a simple visited-site fraction from identifying the national candidate share. EITI workbooks for 13 country contexts describe fiscal institutions; their often earlier vintages and aggregate currency accounts do not identify 2024 commodity-specific collection. (IPIS, 2026; EITI, 2026)

Rough diamonds provide a certification comparison through the Kimberley Process. Carats, values and certificate records are kept separate: quality and sorting prevent direct insertion into the homogeneous-metal model. Copper and nickel describe industrial and coproduct contexts. Their prices and output do not enter the five-mineral equilibrium unless an explicitly named joint-revenue experiment provides the physical and revenue accounts. Observed 2025 updates remain separate from the 2024 calibration. (Kimberley Process, 2026)

Table 1: Reference scales and fixed-value weights, 2024

Mineral	Mine content (t)	Reference USD/t	Weight (%)
Gold	3,661.2	76,718,111	93.39
Cobalt	302,000.0	26,103	2.62
Tantalum	2,500.0	203,915	0.17
Tin	294,000.0	30,159	2.95
Tungsten	82,000.0	31,778	0.87

Notes: Weights are $P_{m,2024}Q_{m,2024}/\sum_j P_{j,2024}Q_{j,2024}$. Gold retains the original WGC scale; other core minerals use the admitted USGS vintage. Prices concern different product stages. The resulting index is an accounting comparison, not mine revenue or welfare.

3 A mineral world with two physical routes

Each observed raw-mineral unit enters either bulk procurement or a retail processing cell. The two routes are disjoint. Eligible and candidate origin classes are assumed strata, not an illegal/legal classification of whole countries. Candidate and eligible extraction sum to raw-mineral extraction; processed output is a different physical object. Resale reallocates a previously extracted unit and does not add another unit of extraction.

The reference extraction technology is quadratic, $C(q) = kq^2/2$. It implies the supply exponent $\eta = 1$ used for the explicit comparison with the extended empirical method. National production and reference prices fix scale and normalization. Buyer values, retention technology, processing

capacities, route shares and acquisition charges remain assumptions. They cannot be jointly estimated from country production totals.

The Muir cell has regulated mass 0.005, regulated value 2 and resale probability 0.02. Its inverse unregulated demand is piecewise, and its exact revenue envelope is globally optimized together with genuine retention. With primary delivery m , retained conversion y , final-good price p and acquisition charge parameter τ , the solved problem is

$$\max_{m,y \geq 0} p\{V_\tau(m) + H_\delta(y)\} - \frac{k}{2}(m+y)^2.$$

Every stationary point, kink, endpoint and exit in the polynomial pieces is compared. The retained conversion uses the source's H_δ construction, $\delta = .001$ and $h_0 = .2$. The reference charge is .05. Actual final output comes from the allocation and resale integrals, not from dividing sales receipts by price.

The VCG cell has two whole owners, each with one package and independent uniform $[0, 1]$ raw productivity. Acquisition charges enter the affected owner's net value. The pivot payment removes the entire owner, not an artificial marginal fraction of that owner. Seller retention is genuine, with value phq , $h = .1$ and $k = .2$. Expected payments, retained stock and physical conversion are evaluated by exact polynomial integrals.

The baseline commits all extracted inventory to the specified mechanism. Making a second indivisible package available can change the receipt schedule discontinuously. The solver compares inventory branches and checks attainment. A supremum approached just below a package boundary is not a valid maximizer. Endogenous withholding is a separately named alternative, not an automatic repair to the all-offered model.

4 Origin claims and three simultaneous clearing equations

The bulk contract retains the canonical funded specification: initial late discovery .25, policy discovery .50, deposit .20, forgery curvature 2, diligence curvature .50, early/late rewards .02 and 1, and fee .02. Fraud and diligence are endogenous; total detection combines policy-set late discovery with endogenous diligence within the declared domains. No-silence, supplier participation and origin backing are checked explicitly.

Let Q_v be vulnerable eligible bulk output, π the fraud fraction and d_π the total detection probability. Diverted candidate mass is $D_v = Q_v\pi/(1-\pi)$; certificate entries are $X = Q_C^{bulk} + D_v$; outside delivery is $Y = Q_D^{bulk} - D_v$. Revoked claims are $d_\pi D_v$ and live candidate claims are $(1-d_\pi)D_v$. A negative outside-delivery stock cannot be rescued by aggregate price clearing.

Let h be the eligible bulk quote, p_D the candidate bulk quote, p the processed-good quote, f the fee and ℓ the expected buyer loss on a vulnerable entry. With actual processed output Z_R the equations are

$$h\{(1+f)X + \ell Q_v/(1-\pi)\} = E_C, \tag{1}$$

$$p_D Y = E_D + \epsilon \frac{Y}{Y + Z_R}, \tag{2}$$

$$p Z_R = E_R + \epsilon \frac{Z_R}{Y + Z_R}. \tag{3}$$

The assumed small coupling is $\epsilon = .001 \min(E_{D0}, E_{R0})$. Reference expenditures fit the baseline at $(h, p_D, p) = (1, .8, 1)$. This exact conditional fit does not identify a demand system. The raw-

mineral world normalizer scales country extraction once; Z_R remains a distinct processed-product unit, not an additional tonne of mineral.

The default static route fraction is .25 for eligible and candidate sources, with .10 and .50 alternatives. The bulk-only compatibility branch removes retail and coupling and separately solves the public-price equations. It is a diagnostic at a different boundary, not an application of the positive-core theorem where its hypotheses no longer hold.

5 The gate, households and three institutions

A mineral restriction reaches extraction through a procurement relationship. Households allocate labor, leisure, consumption and cash subject to outside options and subsistence. The gate can recover part of the cost of goods sold to miners. Selling flexibility, the goods interface and subsistence constraints therefore change the cost of procuring the same output and the response to a charge. The original exact inverse-household branches are retained, while the selling mechanisms are rebuilt from the nonlinear theory.

Each of eight institutional configurations is optimized under the intervention and its own zero-charge comparator. Both exact household cash branches enter the procurement envelope. The reference production grid has 8,001 points and the refinement has 16,001. Margins remain on a declared grid. These are finite hypothetical village economies, not a statistical fit to a national household population.

For Muir, flexible selling chooses primary delivery and genuine retention at each stock. The fixed-route alternative sends .01 of delivery to the regulated group and the rest to unregulated demand, without resale. For VCG, a whole package corresponds to .1 physical units. Its fixed-route comparator posts price .5 to the same two owner distributions, randomly assigns one package, and retains refusals. That comparator differs from the older finite bid-book exercise. This change is a substantive institutional choice and is disclosed when the two workstreams are compared.

6 Paired effects and Shapley attribution

Let $\Delta(S) = q_0(S) - q_1(S)$ be the extraction contraction under configuration $S \subseteq \{s, i, b\}$, where s denotes selling flexibility, i the goods interface and b subsistence. Define absorption $v(S) = \Delta(\emptyset) - \Delta(S)$. The contribution of institution j is

$$\phi_j = \sum_{S \subseteq N \setminus \{j\}} \frac{|S|!(3 - |S| - 1)!}{3!} \{v(S \cup \{j\}) - v(S)\}.$$

Equivalently, average the marginal effect over all six orders. Negative contributions are retained. If the reference contraction vanishes, its absorption ratio is undefined even when the absolute attribution is valid.

The nonlinear design supplies 216 allocations and 18 separate attributions, with 144 paired comparisons. Each mechanism is compared against its own baseline and its own fixed-route construction. Repeating the resulting labels for five minerals indicates applicability, not five independent estimated village economies. Some Muir reference contractions, including .00005 and .00010 normalized units, are of the same order as the production-grid spacing. Their precise magnitudes and exact zero responses must be read with the supplied 16,001-point refinement, not treated as grid-independent estimates.

Table 2: Base village: contraction and channel contributions

Mechanism	τ	Reference	Full	Selling	Goods	Subsistence
Muir	0.05	0.000050	0.000000	0.000025	0.000000	0.000025
Muir	0.1	0.000100	0.000000	0.000050	0.000000	0.000050
VCG	0.05	0.000000	0.000000	0.000000	0.000000	0.000000
VCG	0.1	0.000000	0.000000	0.000000	0.000000	0.000000

Notes: All entries except the charge are normalized mineral quantities. The three contributions sum to the reference contraction minus the full-configuration contraction. Negative contributions are meaningful and are not truncated.

Institutional adjustment can preserve output while shifting losses into gate profits, household payments, retained stock or trading composition. The base full-configuration VCG allocation exits: production is zero in both paired states. Its zero contraction therefore does not demonstrate resilience of positive production. Participation levels must accompany changes. Moreover, the VCG all-offered and posted-price menus are not nested; its selling component is a selling-institution switch, not an unqualified expansion of flexibility. Muir retains the primary-sale and retention choice. A zero extraction response is not proof that no participant is affected. Conversely, a large compositional change in a regulated sales stream need not imply a similarly large mine-output change. The full allocation tables therefore report procurement cost, participation, consumption, cash, profit, retention, sold stock and final output separately.

7 From global routes to empirical exposure

Global trade margins describe where recorded products enter or leave customs territories. The six-country 3T routes provide a narrower bilateral diagnostic for DRC, Rwanda, Nigeria, Indonesia, Bolivia and Myanmar. Their within-origin and composition decomposition should be interpreted only over its reported matched population. A changing origin mix can move the aggregate destination share without a common change in individual routes.

Actual buyer exposure requires the covered product, actor, date and enforcement channel. Trade transshipment, a certificate count and an effective restriction are different objects. HS261590 remains mixed niobium/tantalum/vanadium trade. Mine-site institutional observations identify plausible channels but not the joint household and buyer distribution needed for a country calibration.

The older workstream’s added concave retention is an external gate inventory value under its original auction. Here retention enters the genuine selling institution. Results can therefore agree on the importance of household and outside options while disagreeing on payment incidence and channel ranking. That disagreement is mechanism-dependent; it is not evidence that one country is more resilient without a matched country microdataset.

Table 3: Candidate extraction under stronger discovery: matched curvature

Mineral	Existing method, $\eta = 1$	Nonlinear Muir	Nonlinear VCG
Gold	-0.00888	-0.00666	-0.00666
Cobalt	-0.50559	-0.37916	-0.37915
Tantalum	-0.13541	-0.10155	-0.10155
Tin	-0.13504	-0.10128	-0.10128
Tungsten	-0.13207	-0.09905	-0.09905

Notes: Percentage changes from each model’s own baseline, outside spending fixed. B sends 25% of both source classes to retail; A has the original bulk construction. Unknown country shares are 0.5. These rows match curvature and observed scales, not every institutional primitive.

A Executable source crosswalk and independent reconstruction

Both consolidated theoretical parents are frozen beside this edition, with their hashes and equation labels. The static implementation maps the source objects `nw:cellproblem`, `nw:V`, `nw:retention`, `nw:actualfinal`, `nw:vcgcontinuation`, `nw:retailclearing` and `nw:jointclearing` into exact cell and world accounts. Dynamic ownership and government timing follow the specified subclasses of `dg:prize`, `dg:bellman` and `dg:population`. The readable implementation specification records every numerical primitive, domain, fitted moment and additional finite-equilibrium selection.

The independent static review reconstructs 475 saved states, 36 separate Muir global optimizations and direct whole-owner allocation/removal calculations using 1.44 million buyer-value pairs per tested VCG case. It also compares 150 bulk-only states with the original method at $\eta = 1$. Its 13,695 checks pass without modifying the equilibrium engine. Verification of a numerical example does not estimate its buyer primitives or validate an unrestricted theorem beyond its hypotheses.

The Muir reference extraction, final output and profit are approximately 0.2050386143, 0.1527677063 and 0.0984661155. The VCG reference extracts one package, obtains expected receipts .343 and profit .243, produces .667 physical final units and retains .01 expected mineral units. Receipt units and final-product units are visibly distinct even at these reference points.

For a VCG retention-only/one-package global switch the lower positive price root is

$$p = \frac{k}{h^2} \{R_h - \sqrt{R_h^2 - h^2}\},$$

with competing stocks ph/k and 1. A finite mixture checks equality of their global profits and an aggregate fraction in $[0, 1]$. It does not split an individual owner. Other nonconvex branches retain exact global comparisons and explicit nonattainment diagnostics.

Dynamic verification reads saved paths without importing the solver. It reconstructs physical markets, origins, owner leases and deposit/refund prizes, research conditions, country Bellman choices and transitions, inherited liabilities, grants and the permanent service reserve. Static/research admission is 10^{-8} and government admission 10^{-6} , with distinct grid and horizon checks. All excluded cases remain in the coverage denominator.

B Country scales, assumptions and missing calibration

The following registry is shared by the structural exercises. “Range” means the assumed candidate fractions 0.1, 0.5 and 0.9, not measured illegal or artisanal shares. Existing gold and DRC cobalt fractions retain their original transport qualifications. ROW is the source residual with a fixed 0.5 candidate split, no vulnerable certificate origin and no government. Uganda tantalum and DRC/Burundi/Uganda tungsten remain descriptive BGS cases without a named structural scale. Zero production and missing calibration are distinct. The fuller BGS producer tabulation is supplied as a separate CSV source scenario; its country coverage should not be confused with the narrower gold calibration.

Table 4: Admitted structural production registry

Mineral	Country or residual	Tonnes	Candidate share
Gold	Peru	108.15	0.1873 (transported)
Gold	Colombia	56.70	0.5386 (transported)
Gold	Rest of world and source rounding balance	3,496.35	0.5 (assumed residual)
Cobalt	United States	200.00	Range (assumed)
Cobalt	Australia	4,780.00	Range (assumed)
Cobalt	Canada	3,350.00	Range (assumed)
Cobalt	China	2,000.00	Range (assumed)
Cobalt	Democratic Republic of the Congo	226,000.00	0.1000 (transported)
Cobalt	Cuba	3,450.00	Range (assumed)
Cobalt	Indonesia	35,000.00	Range (assumed)
Cobalt	Madagascar	3,100.00	Range (assumed)
Cobalt	Papua New Guinea	2,630.00	Range (assumed)
Cobalt	Philippines	3,100.00	Range (assumed)
Cobalt	Russia	8,000.00	Range (assumed)
Cobalt	Turkey	2,200.00	Range (assumed)
Cobalt	Rest of world and source rounding balance	8,190.00	0.5 (assumed residual)
Tantalum	United States	0.00	Zero scale; no experiment
Tantalum	Australia	52.00	Range (assumed)
Tantalum	Bolivia	2.00	Range (assumed)
Tantalum	Brazil	210.00	Range (assumed)
Tantalum	Burundi	2.00	Range (assumed)
Tantalum	China	76.00	Range (assumed)
Tantalum	Democratic Republic of the Congo	1,270.00	Range (assumed)
Tantalum	Ethiopia	40.00	Range (assumed)
Tantalum	Mozambique	55.00	Range (assumed)
Tantalum	Nigeria	390.00	Range (assumed)
Tantalum	Russia	29.00	Range (assumed)
Tantalum	Rwanda	374.00	Range (assumed)

Mineral	Country or residual	Tonnes	Candidate share
Tin	United States	0.00	Zero scale; no experiment
Tin	Australia	11,300.00	Range (assumed)
Tin	Bolivia	21,200.00	Range (assumed)
Tin	Brazil	27,600.00	Range (assumed)
Tin	Myanmar	20,000.00	Range (assumed)
Tin	China	71,000.00	Range (assumed)
Tin	Democratic Republic of the Congo	26,000.00	Range (assumed)
Tin	Indonesia	55,000.00	Range (assumed)
Tin	Laos	1,860.00	Range (assumed)
Tin	Malaysia	5,460.00	Range (assumed)
Tin	Nigeria	3,100.00	Range (assumed)
Tin	Peru	32,300.00	Range (assumed)
Tin	Russia	3,260.00	Range (assumed)
Tin	Rwanda	4,100.00	Range (assumed)
Tin	Vietnam	11,000.00	Range (assumed)
Tin	Rest of world and source rounding balance	820.00	0.5 (assumed residual)
Tungsten	United States	0.00	Zero scale; no experiment
Tungsten	Australia	920.00	Range (assumed)
Tungsten	Austria	840.00	Range (assumed)
Tungsten	Bolivia	1,700.00	Range (assumed)
Tungsten	China	67,000.00	Range (assumed)
Tungsten	Kazakhstan	0.00	Zero scale; no experiment
Tungsten	North Korea	1,900.00	Range (assumed)
Tungsten	Portugal	650.00	Range (assumed)
Tungsten	Russia	1,500.00	Range (assumed)
Tungsten	Rwanda	1,300.00	Range (assumed)
Tungsten	Spain	700.00	Range (assumed)
Tungsten	Vietnam	3,400.00	Range (assumed)
Tungsten	Rest of world and source rounding balance	2,090.00	0.5 (assumed residual)

C Measurement and procurement admission

The oxide conversion for tantalum uses the declared Ta fraction 0.81897 in Ta_2O_5 . A quotation of USD 167 per kg oxide-equivalent therefore gives $167000/0.81897$ USD per tonne of contained Ta. Tungsten uses the declared 0.793 W fraction in WO_3 : one dry metric tonne unit is 10 kg WO_3 , or 7.93 kg W, giving $252000/7.93$ USD/t W for a USD 252/dmtu reference. Cobalt and tin pound quotations use 0.45359237 kg/lb. All these conversions are checked separately from their suitability as procurement prices.

The Rwanda 3T study provides evidence of assays, payment stages and food deductions, but its tantalum table is not admitted as an elasticity moment. The COMIKAGI table reports 1,540 kg concentrate per month at 30% Ta₂O₅, a USD 130/kg quotation of unclear basis, and USD 37.7/kg concentrate locally. The displayed international total USD 200,200 treats the quotation as per kg concentrate; an oxide-content application instead gives USD 60,060. The table’s combined total USD 514,578 also differs from USD 45,619 plus USD 200,200. The local total USD 58,058 is internally consistent. These alternatives imply radically different ratios, so no tantalum procurement elasticity is inferred. ([Rwanda study, 2018](#))

The cassiterite example is arithmetically coherent: 3,960 kg concentrate, 60% tin and USD 19.20/kg metal imply approximately USD 45,619 internationally; USD 11/kg concentrate implies USD 43,560 locally. The ratio is approximately 0.955. Gifurwe’s tungsten comparison gives approximately 0.686 from USD 8.4/kg concentrate and USD 188.5/dmtu at 65% WO₃. Both are transaction-stage ratios, not supply elasticities. The independently inspected pages and all arithmetic are retained in the Rwanda procurement audit.

Observed scales, conditional fits and assumptions have distinct records. No new mineral inherits the Colombia or Peru procurement elasticity merely because it is artisanal or traded through the same region. Identifying that elasticity requires matched transaction prices, assay and deduction data, production costs, net margins and a defensible population bridge.

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